Southwest Power Pool

DYNAMIC OPPORTUNITY COST MITIGATED ENERGY OFFER FRAMEWORK FOR ELECTRIC STORAGE RESOURCES

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By: Market Monitoring Unit

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INTRODUCTION

A mitigated energy offer for a generating resource reflects the short-run marginal cost of production for the resource. For a typical generating resource, the calculation of the short-run marginal production cost derives from variables including the incremental heat rate and fuel cost (where applicable), and variable operations and maintenance cost [5]. When the ability for a resource to operate is limited within a period of time, the short-run marginal cost may also include opportunity cost associated with incremental generation. Opportunity cost may be appropriately included if incremental generation at a given point can only be accomplished by forgoing profits associated with a future opportunity.

In the case of an electric storage resource (ESR), generating or charging at a given point in time may only be possible by forgoing profit opportunities later in the day or optimization period. When this marginal opportunity cost exists, it is appropriate to include in the basis for an ESR mitigated energy offer. The marginal opportunity cost of an ESR at any point in time is most accurately determined as the result of a dynamic optimization problem that considers the resource characteristics, state-of-charge, and all future profit opportunities in the optimization period[1]. In practice, this may be difficult or impractical to implement for purposes of calculating a mitigated energy offer. A reasonable approximation of this opportunity cost can be determined for ESRs with relatively short charge and discharge times by considering a simplified case to establish a lower bound of expected profits. This lower bound is represented by the expected maximum profit, or maximum profit that would be earned if actual prices were realized as predicted. The approximation of marginal opportunity cost can then be determined by assessing the reduction in this expected maximum profit that may result from operating at a given point in time[2]. When implemented, this approach allows for multiple economic charge and discharge opportunities as actual prices are realized that account for the potential opportunity cost of any reduction in the lower bound of expected profit.

SUMMARY OF APPROACH

An ESR with an operating range of positive or negative output can be thought of as a generator that either produces energy using the input or fuel of charging energy or produces the equivalent of incremental generation by reducing the level of charging at a point in time. As indicated in a number of other works on this topic (for example, [1],[2],[4]) , profit maximization for the ESR operator results from a dynamic optimization problem where actions taken in one interval of the

[1] An ESR's state-of-charge represents the percent of energy stored in the resource in relation to the amount of energy that can be stored. State-of-charge provides an indicator of the ESR's ability to charge or discharge at a point in time.

[2] Formally, this results from including opportunity cost as part of the total production cost for the ESR, and taking the first derivative to find marginal cost. The standard economics definition of marginal cost is the first derivative of a producer's total cost function with respect to output: $\frac{\partial TC\ (Q)}{\partial Q}$ = MC(Q) [3]. Intuitively, this is the amount by which total cost changes with an incremental change in output.

optimization period may impact the set of possibilities and outcomes in a later interval. The profit maximizing choice in a given period is dependent upon expectations of energy price (i.e., the locational marginal price or LMP), the operating capabilities or limitations of the resource, and the state-of-charge at that point in time. In the case of an ESR that can fully charge by operating at maximum charge for one interval of an optimization period, and fully discharge by discharging at full output for one interval of a period, the problem is simplified. The ESR will maximize profit from energy sales from a single charge and discharge when the resource maximizes the price spread between the LMP it pays for charging energy and the LMP it gets paid for discharging energy, accounting for roundtrip efficiency losses[3]. In a period defined by a single pricing trough (local minimum) and single pricing peak (local maximum), the expected profit is greatest when the ESR charges at the pricing trough and discharges at the pricing peak. Over a longer optimization period (e.g., an operating day or multiple operating days), there may be multiple peaks and troughs in prices. These multiple troughs and peaks can be evaluated to establish the optimization sub-periods associated with the expected maximum profit. The expected profit for the longer optimization period is maximized when the expected profits of each defined optimization sub-period are maximized.

Although the expected maximum profit from energy sales is achieved by charging at appropriate pricing troughs and discharging at sufficiently high pricing peaks, the exact timing of when such prices will occur is unknown. The ESR operator is dependent on a price forecast over the longer optimization period to form an expectation of the overall profit maximizing operation of the ESR[4] [1],[2],[4]. Earlier work on the subject of energy arbitrage opportunities for ESRs notes that perfect foresight of arbitrage opportunities is impossible [1],[2],[4]. However, with a robust price forecast, a bidding strategy constructed around the forecast of profit maximizing operation can result in realizing a significant portion of the expected maximum profit [2]. These findings provide general support to the idea that expectations of profit maximizing outcomes can be used to inform the opportunity cost of the ESR charging or discharging at other points in time. In the approximation presented here, the marginal cost at each point (e.g., hour) of the optimization period will incorporate the opportunity cost that would be incurred at that point if the resource were to deviate from the operating plan that would achieve the maximum expected profit. This marginal cost can be an input to a mitigated energy offer. At each point on the expected price curve, the mitigated offer incorporates opportunity cost such that the resource will not be economic to dispatch if doing so would make the ESR operator realize profits less than the lower bound of maximum expected profit. The ESR would only be economic if the actual price differs from the forecasted price and operation would not result in a reduction of maximum expected profit.

The output or charging capability of an ESR is limited in duration and it is in the economic interest of the ESR operator to allocate the charging and discharging of energy in a profit maximizing manner. Thus, like some other resource types (e.g., hydro with pondage or thermal resources with run-hour air permit restrictions), the concept of opportunity cost applies to ESRs: if the limited duration of energy production is used before the profit maximizing opportunity (hours), some

[3] Roundtrip efficiency losses represent energy lost between charging and discharging. For example, a storage resource that charges 1 MW and discharges 0.9 MW would have roundtrip efficiency losses of 10%. The roundtrip efficiency factor for this resource would be 0.9 or 90%.
[4] The forecasting of hourly price values is not discussed here. The general framework is flexible and could accommodate prices derived from any forecasting or approximation method deemed appropriate.

amount of profit is foregone. Similarly, for an ESR that faces a reduction in charging, profit may also be foregone when charging at a point in time provides a reduction in future interval charging cost associated with the maximum expected profit.

ESRs that are limited in their ability to discharge or recharge due to time or physical limitation may be analogous to other fuel-limited or use-limited resource types. However, there are some key differences of ESRs that can charge or discharge more frequently. First, for an ESR positioned to discharge, the resource may have the potential to discharge and charge again before reaching the peak price hour associated with minimum expected profit. In other words, if a recharging opportunity exists, an "early" discharge before reaching the expected maximum profit peak price operating hour(s) does not preclude the ESR from also producing in that peak price hour and realizing the associated profits. It only implies that some charging energy would have to be replaced at a potentially higher price than that paid for the initial charging energy, and thus the profit from discharge in the next peak price hour would be reduced, but not entirely eliminated. Additionally, ESRs modeled as generators may have an offer curve that extends into the charging (negative output) range of the resource. In this range, a reduction in ESR charging is equivalent to incremental generation. Therefore, when considering the mitigated energy offer for a generator, the appropriate marginal cost associated with this operating range must also be contemplated as the marginal cost of charging reduction. Resources with local market power in generation can potentially exercise this market power by charging at uneconomically high prices. An ESR with local market power in generation that continues to charge when uneconomic has the same market impact as a generator with local market power withholding incremental generation.

The case presented here is the simplified case of a resource capable of full charge or discharge by operation at full output or consumption in single interval of an optimization period. Resources which do not operate in this way face a more complex dynamic optimization problem to determine expected maximum profit. This problem is more dependent on the state-of-charge at a given point in time and the number of remaining intervals in the optimization period. For these resources, the simpler case presented here implicitly assumes the resource is always positioned for its last available interval of charge or discharge and would face opportunity cost associated with the highest valued future profit opportunity. For resources that require relatively few intervals of operation at full output or consumption within the optimization period to fully charge or discharge, the approach presented here may still be an appropriate basis for a mitigated offer. As more intervals operating at maximum charging or output are required to fully charge or discharge, the estimate is potentially less accurate as accuracy becomes more dependent on state-of-charge and the point in the optimization period. For long duration ESRs, depending on state-of-charge, the opportunity cost associated with actions in any one interval may diminish and marginal cost approaches the cost of fuel and other incremental production cost.

The sections below present an ESR mitigated energy offer framework that considers opportunity cost in the context of potential reductions to an expectation of maximum profit which would occur if prices were realized as forecasted. This framework considers a resource that can fully charge or discharge in one optimization interval (e.g., hour) of full output operation. The first section outlines the maximum expected profit outcome for the ESR as defined by expected prices. This calculation is used to form the basis for opportunity cost at different points on the expected price curve. The following three sections define the profit and cost (inclusive of opportunity cost) of charging and discharging energy at different points on the expected price curve over the optimization period.

The total cost of charging or discharging energy is then used to establish the marginal cost[5] of discharging or reducing charging at that point on the expected price curve over the optimization period. The final section presents a summary table that could be useful in a practical application of the mitigated offer methodology.

SECTION 1: ESTABLISHING THE EXPECTED MAXIMUM PROFIT FOR ESR FROM ENERGY SALES

Consider an ESR that is capable of full charge or full discharge by operating at maximum charge or maximum output for a single interval of a defined optimization period. For a given optimization period containing N optimization sub-periods, expected maximum profit is defined by the following equation where charging occurs at the trough and discharging occurs at the peak[6,7]:

$$E(\pi_{MAX,i}) = \sum_{i=1}^{N}(Y_i - X_i /L)*Q_i$$

Where:

X_i = the expected trough price for optimization sub-period i

Y_i = the highest expected peak price for optimization sub-period i such that $Y_i \geq X_{i+1}/L$

L = roundtrip efficiency factor of ESR, (1- roundtrip loss percentage), and $0 \leq L \leq 1$

Q_i = quantity of energy discharged at the peak of optimization sub-period i

Therefore the total expected maximum profit for the optimization period is maximized when the expected profit of each optimization sub-period i is maximized. Importantly, as indicated by the definition of Y_i , not every peak and trough on the expected price curve will define a optimization sub-period associated with maximum expected profit. Consider the case of any two peaks (Y_k , Y_{k+1}) and two troughs (X_k , X_{k+1}).

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[5] The term "marginal cost" as referenced in this document is always intended to mean short-run marginal cost.

[6] As described above, expected maximum profit should be interpreted throughout this document as the lower bound of profit expected over the optimization period or sub-period, i.e., that which would be realized if prices occurred exactly as forecasted.

[7] A formal proof is not presented here, however, the result can be shown for a number of different examples when prices are consistently increasing or decreasing. The result derives from solving a dynamic optimization model like that presented in [1], and the more general model could potentially serve as a basis to formally prove the result.

In this case:

$$\text{If } E(\pi) = (Y_k - X_k/L)*Q + (Y_{k+1} - X_{k+1}/L)*Q \geq E(\pi) = (Y_{k+1} - X_k/L)*Q$$

$$\Longrightarrow (Y_k - X_k/L) + (Y_{k+1} - X_{k+1}/L) \geq (Y_{k+1} - X_k/L)$$

$$\Longrightarrow Y_k + (Y_{k+1} - X_{k+1}/L) \geq Y_{k+1}$$

$$\Longrightarrow Y_k \geq X_{k+1}/L$$

When the condition $Y_k \geq X_{k+1}/L$ is satisfied, the pair of peaks and troughs will define two optimization sub-periods, over which maximizing profits of each will maximize total expected profit for the optimization period. However, if the condition is not satisfied, only one optimization sub-period will be defined, with a profit maximizing peak at Y_{k+1} and a trough at the least cost charging opportunity before that peak, $\min(X_k, X_{k+1})$.

Following this logic and using the expected prices for the optimization period, an ESR may establish the maximum expected profit based on a price forecast over the optimization period. Peaks and troughs on the expected price curve for the optimization period which satisfy the profit maximizing criteria above will define the optimization sub-periods. In order to establish the number of optimization sub-periods and expected maximum profit, each peak-to-peak area on the expected price curve must be evaluated iteratively. For example consider a price curve containing two peaks. The first peak encountered on the expected price curve, Y_1, is evaluated against the next trough, X_2. If $Y_1 \geq X_2/L$, the peak Y_1 is a profit maximizing peak. Then there are two optimization sub-periods in which maximizing profit of each will result in the maximum profit of the optimization period: charging at X_1, discharging at Y_1 and charging at X_2, discharging at Y_2.

Now consider the case where, for the same peak-to-peak evaluation, $Y_1 < X_2/L$. In this case, Y_1 is not a profit-maximizing peak and there is only one optimization sub-period with peak at Y_2. The trough defining this period for which maximum profit is expected is the minimum of X_1 and X_2, the troughs preceding Y_1 and Y_2. The iterative process would continue if there were a third trough (X_3) and peak (Y_3) occurring after Y_2. Trough X_3 would be compared to peak Y_2. If $Y_2 \geq X_3/L$, then there would be two optimization sub-periods: one defined by trough value $\min(X_1, X_2)$, and peak Y_2, and one defined by trough X_3 and peak Y_3. If $Y_2 < X_3/L$, there would still be only one optimization sub-period with trough defined by $\min(X_1, X_2, X_3)$ and peak defined by Y_3. This iterative process would continue to evaluate all peak-to-peak points on the expected price curve of the optimization period, and would form the basis for the expected maximum profit.

The ESR operator forms an expectation of maximum profit for the optimization period based on expected prices over the optimization period. Once optimization sub-periods are identified, the ESR operator can identify where each interval is in relation to expected profit maximizing peaks and troughs. As actual market prices are realized, the profit maximizing ESR should be willing to charge and discharge at different points in time on the condition that doing so does not cause realized profits to be less than the expected maximum profit for the optimization sub-period. The cost-based offer in each hour of the optimization period will reflect the opportunity cost of lost expected profit that could result from operation in a manner that departs from that required to achieve the expected maximum profit[8]. This value is dynamic throughout the optimization period and is dependent on how the expected value of the given hour relates to the expected profit maximizing peaks and troughs of the optimization period.

The scenarios below illustrate how the total cost of producing charging or discharging energy can evolve over the optimization period, and thus how the marginal cost basis of the mitigated offer can also evolve. While the logic for the appropriate mitigated energy offer at each point of the expected optimization period price curve may vary, the answer generally is derived from the expected price in the next operating interval (e.g., hour) of the optimization period, adjusting for round trip efficiency losses as appropriate. The following scenarios all assume that ESRs can charge and discharge multiple times before reaching a profit maximizing peak price or trough price within an optimization sub-period, limited only by the amount of time remaining before reaching the peak or trough. Each sub-period is defined as beginning when the resource is discharged following a profit maximizing peak with prices moving toward the next expected trough associated with profit maximization, and ending with the next discharge of the resource at next expected profit maximizing peak.

SECTION 2: HOUR IMMEDIATELY PRECEDING EXPECTED PROFIT MAXIMIZING PEAK, OR EXPECTED PRICES MOVING TOWARD EXPECTED TROUGH ASSOCIATED WITH PROFIT MAXIMIZATION

Consider the point on the optimization period expected price curve where prices are moving toward an expected trough associated with profit maximization or immediately preceding an expected profit maximizing peak. This can include the hour immediately preceding an expected profit maximizing peak, the hour of an expected profit maximizing peak, or the hours following a peak up to but not including the hour immediately preceding the next expected trough associated with profit maximization.

CHARGING

Assume the ESR is physically positioned to receive a charge before reaching the next expected trough in optimization sub-period i.

[8] The cost-based mitigated energy offer may also include a variable operations and maintenance cost adder if applicable [5].

Assuming that charging occurs at the optimization sub-period trough and discharging occurs at the peak, the expected maximum profit for optimization sub-period i is:

$$E(\pi_{MAX,\,i}) = (Y_i - X_i/L)*Q_i$$

Where:

X_i = the expected trough price for optimization sub-period i

Y_i = the highest expected peak price for optimization sub-period i such that $Y_i \geq X_{i+1}/L$

L = roundtrip efficiency factor of ESR, (1- roundtrip loss percentage), and $0 \leq L \leq 1$

Q_i = quantity of energy discharged in optimization sub-period i

Defining charging and discharging energy as distinct products and decomposing the expected maximum profit equation to the relevant expected revenue and cost, the expected maximum profit for optimization sub-period i can be equivalently written as:

$$E(\pi_{MAX,\,i}) = E(REVENUE\pi_{MAX,\,i}) - E(COST\pi_{MAX,\,i})$$

$$= Q_{DC,i}*Y_i - Q_{CG,i}*X_i$$

Where:

$Q_{DC,i}$ = Quantity of energy discharged in optimization sub-period i

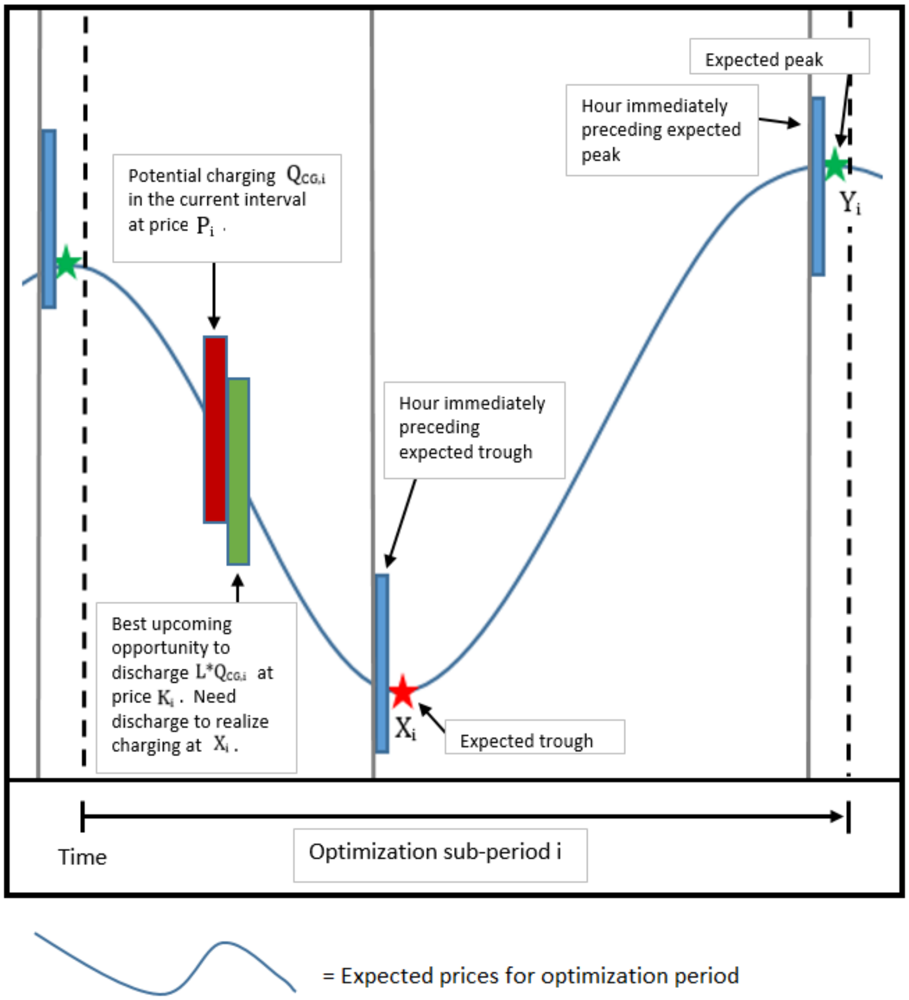
$Q_{CG,i}$ = Quantity of energy charged in optimization sub-period i = $(Q_{DC,i}/L)$

$Q_{DC,i} \geq 0$, $Q_{CG,i} \geq 0$

and X_i , Y_i are defined as above.

Assume that there exists an additional discharge opportunity at expected price $K_i > X_i$ before reaching the expected trough, and an additional charging opportunity occurring in the hour before the expected price of K_i where the realized price is P_i. In this situation, the ESR could charge $Q_{CG,i}$ at price P_i and discharge quantity L* $Q_{CG,i}$ at price K_i before reaching the interval of X_i, thereby retaining the opportunity to again charge at the expected profit maximizing trough price X_i. If charging occurs at P_i, an additional discharge opportunity is created. At this point on the expected price curve, the highest valued discharge opportunity would be expected in the next hour at price K_i. The cost or revenue associated with discharging $L^* Q_{CG,i}$ can be thought of as a reduction or addition to the expected profit maximizing charging cost. This situation is summarized in Figure 1.

Figure 1



Under this scenario, if charging occurred at P_i, cost associated with the expected maximum profit in optimization sub-period i would change as follows as a result of the additional discharge opportunity:

$$\Delta E(COST\pi_{MAX,i}) = (Q_{CG,i}{*}X_i - L^* K_i{*} Q_{CG,i}) - Q_{CG,i}{*}X_i$$

$$= - L^* K_i{*} Q_{CG,i}$$

In the case of an ESR that is charging, it would be more common and intuitive to think of the charging energy as in input to production of discharging energy, and to identify the ESR operator's willingness to pay for an additional unit of that input. However, given that ESRs may be modeled as generators with a continuous operating range capable of producing negative output (i.e. charging when modeled as a generator), the discussion here considers ESR charging in the framework of a producer of that output. In this framework, a reduction of produced charging energy is equivalent to incremental generation. The applicable marginal cost to estimate is that of incremental generation. Therefore, for the charging range of an ESR modeled in a generator framework, the appropriate marginal cost is that associated with a reduction in charging energy production. Potential opportunity cost associated with profit changes from $E(\pi_{MAX,i})$ are appropriately included as a cost of reduced charging energy in a given interval.

Total economic profit from producing any output can be expressed generally as[9]:

$$\Pi = TR - TC$$

$$= TR - FC - VC - OC$$

$$= TR - [\, FC + VC + OC \,]$$

Where:

TR = Total revenue

TC = Total cost

FC = Fixed cost

VC = Variable cost

OC = Opportunity cost

The general profit form can be applied to the more specific charging scenario on the portion of the expected price curve described above. This can be useful to identify total cost. Because the production of charging energy reflects negative output of the resource, and to more easily contemplate decreases in charging as incremental generation, consider profit as a function of variable $D_{CG,i} = - Q_{CG,i}$. A one unit increase in $D_{CG,i}$ is equivalent to a one unit reduction in charging or a one unit increase in generation. In the profit equation, revenue (negative or positive) is realized from producing charging energy, there is no variable cost from fuel, and the change in cost

[9] Economic profit includes accounting costs as well as opportunity costs. This is differentiated from accounting profit which does not account for opportunity costs.

$\Delta E(COST\pi_{MAX,i})$ is realized if charging occurs at price P_i[10]. Note that this value can be positive or negative.

Consider $\Delta E(COST\pi_{MAX,i})$ as OC_i in the production of charging energy. When defined as a function of $D_{CG,i}$, this can be interpreted as the opportunity cost associated with incremental generation (a decrease in charging) on the charging output range. When charging is decreased, this is the potential reduction in expected profit maximizing charging cost that is foregone.

Then total profit realized from charging in an interval of sub-optimization period i is:

$$\Pi(D_{CG,i}) = P_i * D_{CG,i} - [FC_i + 0 + \Delta E(COST\pi_{MAX,i})]$$

$$= P_i * D_{CG,i} - [FC_i + L^* K_i * D_{CG,i}]$$

Total cost of charging on this portion of the expected price curve in an interval of sub-optimization period i is:

$$TC(D_{CG,i}) = FC_i + L^* K_i * D_{CG,i}$$

Taking the first derivative of the total cost function TC with respect to $D_{CG,i}$ yields MC , the marginal cost of a decrease in charging (or equivalently incremental generation) in an interval of sub-optimization period i at the specified point of the expected price curve:

$$\frac{\partial TC}{\partial D} = MC = L^* K_i$$

The value $L^* K_i$ forms the basis for the mitigated energy offer for the charging operating range at this point on the expected price curve, for the hour before expected price K_i. This value should be interpreted from the perspective of a generator operating in the charging range, where a reduction in charging (a less-negative value of $D_{CG,i}$) is equivalent to incremental generation. This is the amount by which $E(\pi_{MAX,i})$ would decrease if charging were occurring and reduced, analogous to incremental generation. Production of charging energy (a negative output) will only be economic to produce when $P_i \leq L^* K_i$. At prices above $L^* K_i$, the marginal cost of reducing charging energy, it

[10] Additional variable costs such as variable operations and maintenance could also be appropriately included here as applicable. These other costs are not shown here to simplify and focus on the opportunity cost element.

becomes economic for the ESR to reduce charging energy. This is analogous to increasing generation output. Producing charging energy at a higher price than $L^* K_i$ would result in a decrease of $E(\pi_{MAX,i})$ [11]. This is because at this point, the potential reduction in expected profit maximizing charging cost will not exceed the (potentially negative) revenue from charging at P_i. If the ESR has local market power in generation, this market power may be exercised if the ESR continues charging (withholds incremental generation) at price above $L^* K_i$.

DISCHARGING

Consider the same point on the optimization period expected price curve where prices are moving toward an expected trough associated with profit maximization or immediately preceding an expected profit maximizing peak. This can include the hour immediately preceding an expected profit maximizing peak, the hour of an expected profit maximizing peak, or the hours following a peak up to but not including the hour immediately preceding the next expected trough associated with profit maximization. However, now consider the ESR as positioned to discharge.

Again, assuming that charging occurs at the optimization sub-period trough and discharging occurs at the peak, the expected maximum profit for optimization sub-period i is:

$$E(\pi_{MAX,\,i}) = E(REVENUE\pi_{MAX,\,i}) - E(COST\pi_{MAX,\,i})$$

$$= Q_{DC,i}{*}Y_i - Q_{CG,i}{*}X_i$$

Also, recall that the expected profit for the optimization period is maximized by maximizing the expected profit for each optimization sub-period i.

If the ESR is positioned to discharge because the expected profit maximizing peak price has not yet been realized (hour immediately preceding an expected profit maximizing peak), the charging cost has been realized and if discharged, there is not sufficient time to recharge before reaching the expected peak price for optimization sub-period i[12]. A discharge of the ESR before reaching the

[11] Although framed in the context of producing charging energy as an output, this result is consistent with the outcome that would be expected when considering charging energy as an input to the production of discharging energy. The profit maximizing producer of discharging energy would be willing to pay up to the marginal revenue expected from discharging that energy [3].

[12] In real-time, expectations of peaks and troughs may be evolving as prices are realized through the day. For instance, consider the case where the initial expected peak price hour passes with a lower than expected realized price, the resource is still charged, and prices are expected to continue declining until the next expected trough. Then the next operating hour will become the new expected peak for optimization sub-period i and the same logic applies.

expected peak hour of optimization sub-period i results in full foregone profit associated with discharge at the expected peak price Y_i. Possible occurrences of this situation are summarized in Figures 2 and 3.

Figure 2

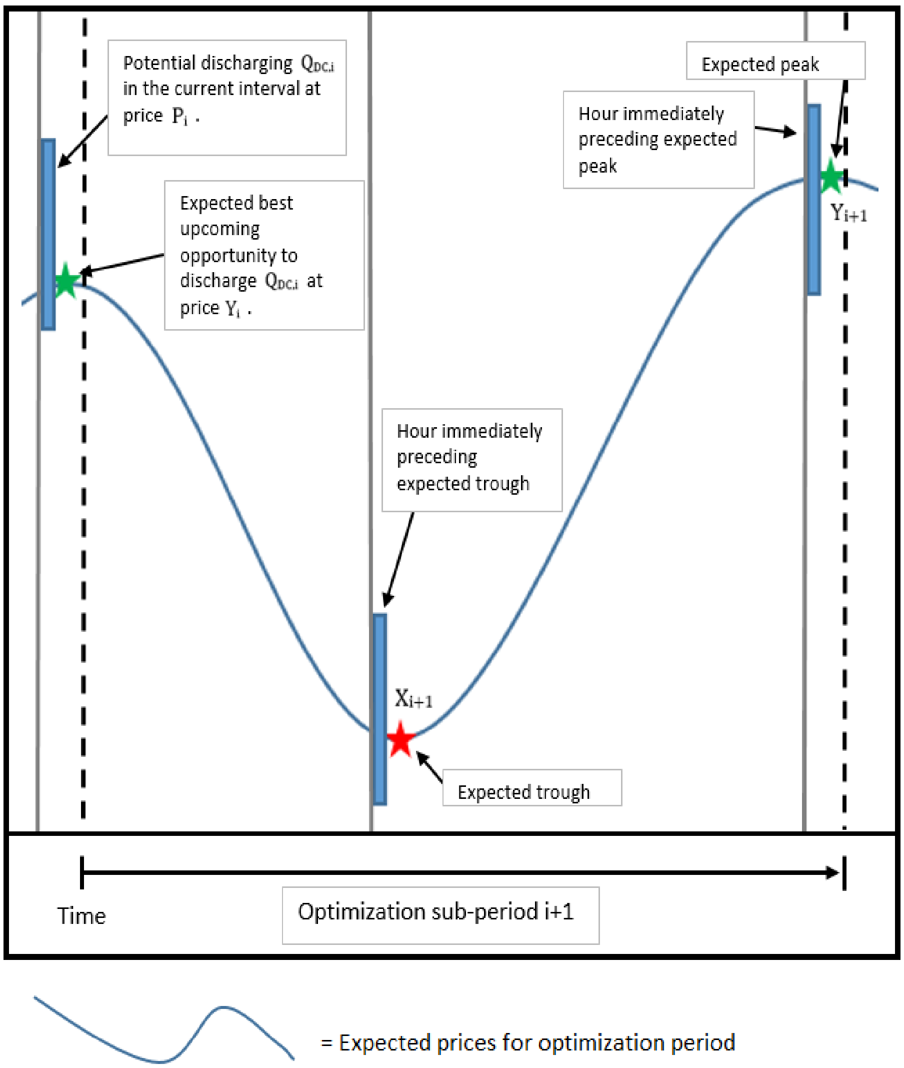
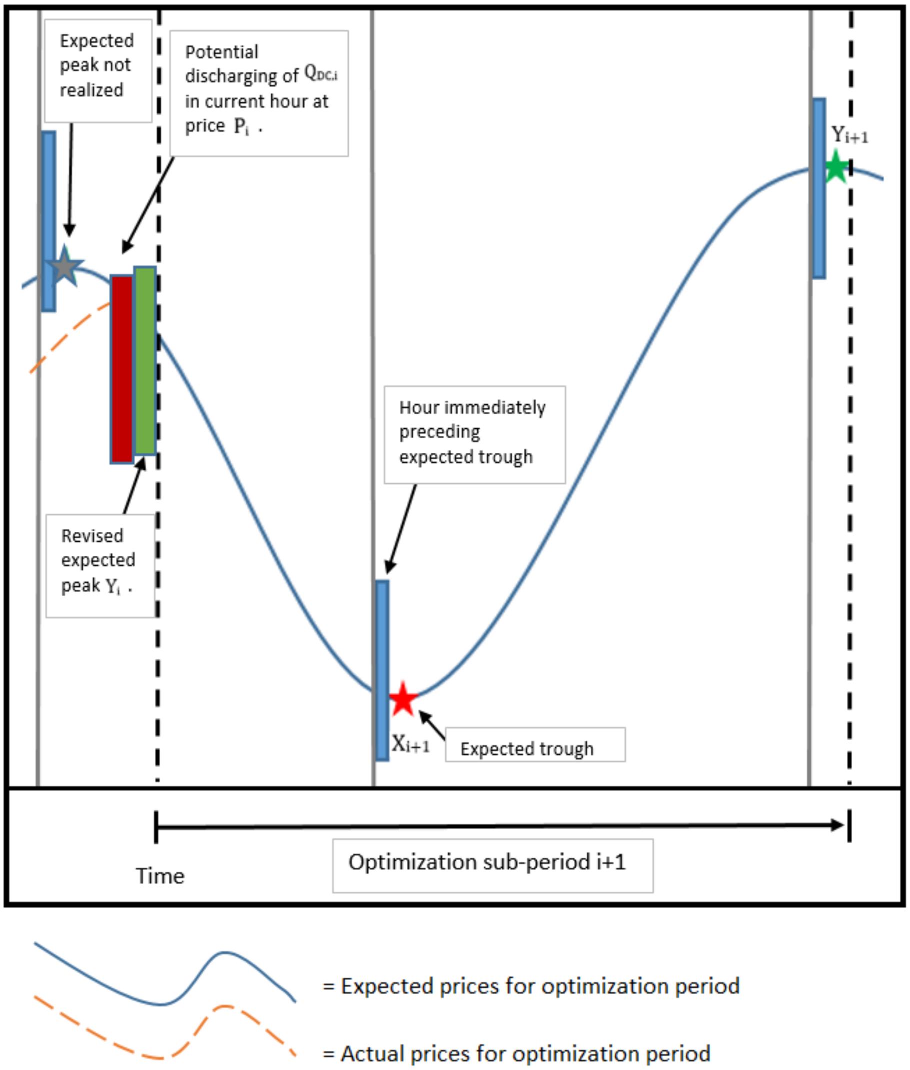


Figure 3



The foregone profit for optimization sub-period I, which represents the opportunity cost from discharging before reaching Y_i when there exists no recharging opportunity, is:

$$\Delta E(\pi_{MAX,i}) = -(Q_{DC,i} * Y_i - Q_{CG,i} * X_i)$$

The value $\Delta E(\pi_{MAX,i})$ reflects that the expected maximum profit is foregone if generating positive output at this point on the expected price curve. It is being given up for profit associated with discharging at the current interval.

Noting that $\Delta E(\pi_{MAX,i})$ represents opportunity cost of foregone profit, and considering the negative of this value to properly interpret as a cost, and noting that the fuel cost can be expressed as $Q_{CG,i}*X_i$, these values can be applied to the general profit formation presented in the discussion of charging cost for an interval of optimization sub-period i as:

$$\Pi(Q_{DC,i}) = P_i * Q_{DC,i} - [FC_i + (-\Delta E(\pi_{MAX,i}))]$$

$$= P_i * Q_{DC,i} - [FC_i + Q_{CG,i}*X_i + (Q_{DC,i}*Y_i - Q_{CG,i}*X_i)]$$

The total cost of discharging on this portion of the expected price curve in an interval of sub-optimization period i is:

$$TC\,(Q_{DC,i}) = FC_i + Q_{CG,i}*X_i + (Q_{DC,i}*Y_i - Q_{CG,i}*X_i)$$

$$= FC_i + Q_{DC,i}*Y_i$$

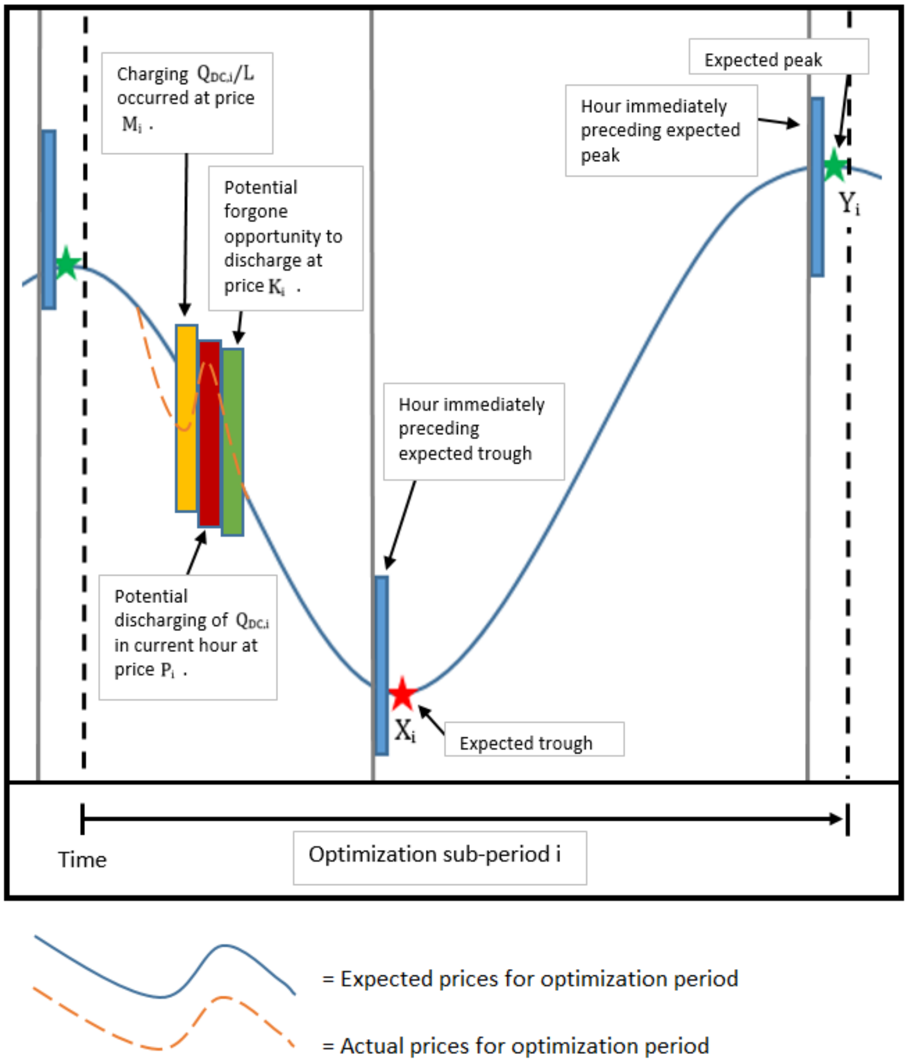
Taking the first derivative of the total cost function TC with respect to $Q_{DC,i}$ yields MC , the marginal cost of discharging in an interval of optimization sub-period i at the specified point of the expected price curve:

$$\frac{\partial TC}{\partial Q} = MC = Y_i$$

This value represents the marginal cost and basis of the mitigated offer for discharging at this point of the expected price curve. The value includes fuel cost for discharging and opportunity cost of potentially foregone profit. For discharging at this point on the expected price curve when the expected profit maximizing peak Y_i is not yet realized, the value corresponds to the latest expected profit maximizing peak price. This is the next expected price K_i, so this scenario is the case of $K_i = Y_i$. However, the result more generally is the expected price in the next hour, K_i.

Now consider the same point on the expected price curve, but consider the scenario that the ESR had discharged at the end of the previous optimization sub-period, recharged at a price M_i occurring after the end of the last optimization sub-period but before the current interval with price P_i, and is again positioned to discharge. At this point, the additional discharge would be required in order to realize the expected profit maximizing charging opportunity at X_i. Considering the current interval, if the ESR discharges, it foregoes the opportunity to discharge in the next interval K_i and any associated profits. This situation is summarized in Figure 4.

Figure 4



The expected maximum profit for optimization sub-period i would change as follows from a discharge in the current interval:

$$\Delta E(\pi_{MAX,i}) = Q_{DC,i}{*}Y_i - Q_{DC,i}/L{*}X_i - (Q_{DC,i}{*}Y_i - Q_{DC,i}/L{*}X_i + Q_{DC,i}{*}K_i - Q_{DC,i}/L{*}M_i)$$

$$= -\,Q_{DC,i}{*}K_i + Q_{DC,i}/L{*}M_i$$

The value $\Delta E(\pi_{MAX,i})$ represents opportunity cost of the additional profit opportunity foregone.

Considering the negative of this value to properly interpret as a cost, and that the fuel cost at which the ESR last charged can be expressed as $Q_{DC,i}/L*M_i$, these values can be applied to the general profit formation presented in the discussion of charging cost for an interval of optimization sub-period i as:

$$\Pi(Q_{DC,i}) = P_i * Q_{DC,i} - [FC_i + (-\Delta E(\pi_{MAX,i}))]$$

$$= P_i * Q_{DC,i} - [FC_i + Q_{DC,i}/L*M_i + (Q_{DC,i}*K_i - Q_{DC,i}/L*M_i)]$$

The total cost of discharging on this portion of the expected price curve in an interval of sub-optimization period i is:

$$TC\,(Q_{DC,i}) = FC_i + Q_{DC,i}/L*M_i + (Q_{DC,i}*K_i - Q_{DC,i}/L*M_i)$$

$$= FC_i + Q_{DC,i}*K_i$$

Taking the first derivative of the total cost function TC with respect to $Q_{DC,i}$ yields MC, the marginal cost of discharging in an interval of optimization sub-period i at the specified point of the expected price curve:

$$\frac{\partial TC}{\partial Q} = MC = K_i$$

As in the other discharge case on this portion of the expected price curve, the marginal opportunity cost associated with discharge to be included in the mitigated energy offer is the expected price in the next hour, K_i.

SECTION 3: HOUR IMMEDIATELY PRECEDING EXPECTED TROUGH ASSOCIATED WITH PROFIT MAXIMIZATION, OR EXPECTED PRICES MOVING TOWARD EXPECTED PROFIT MAXIMIZING PEAK

Consider the point on the optimization period expected price curve where prices are moving toward an expected peak or immediately preceding an expected trough associated with expected profit maximization. This can include the hour immediately preceding an expected trough, the hour of an expected trough, or the hours following a trough up to but not including the hour immediately preceding the next expected profit maximizing peak.

DISCHARGING

As defined in previous sections, assuming that charging occurs at the optimization sub-period trough and discharging occurs at the peak, the expected maximum profit for optimization sub-period i is:

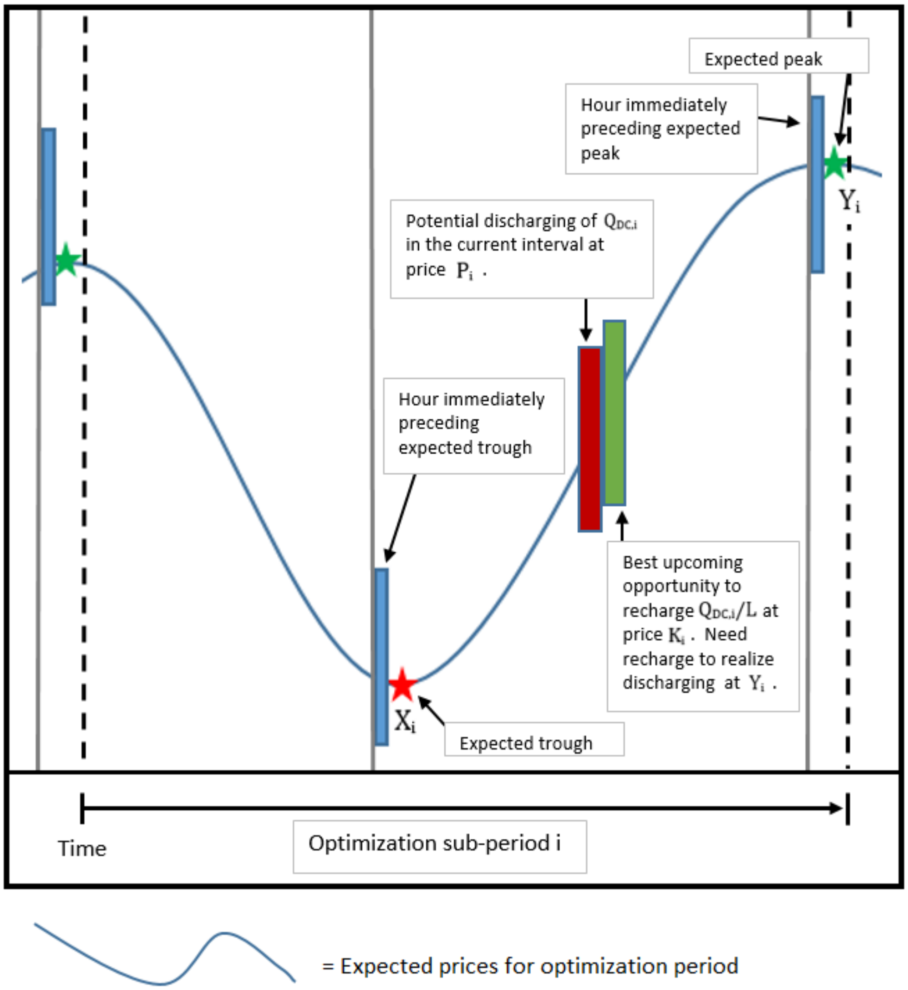
$$E(\pi_{MAX,\,i}) = E(REVENUE\pi_{MAX,\,i}) - E(COST\pi_{MAX,\,i})$$

$$= Q_{DC,i} * Y_i - Q_{CG,i} * X_i$$

$$= Q_{DC,i} * Y_i - Q_{DC,i} / L * X_i$$

If the resource is positioned to discharge, the expected profit maximizing discharge opportunity at this point on the optimization period expected price curve will be at price Y_i. However, for hours where Y_i is not expected to occur in the next hour, if the ESR discharges in the current hour at a price P_i, the ESR will have an opportunity to recharge at price K_i occurring before Y_i. Therefore, the opportunity to discharge at the profit maximizing price Y_i is not foregone. However, the realized profit would be lower than $E(\pi_{MAX,\,i})$ as replacement charging energy to produce at Y_i would be purchased at price $K_i > X_i$. Further, the amount of replacement charging energy that must be purchased at K_i to later produce the same output at Y_i is 1/L times the amount discharged at P_i to account for roundtrip efficiency losses. This situation is summarized in Figure 5.

Figure 5



The change in the expected maximum profit opportunity for optimization sub-period i when an early discharge results in the purchase of replacement charging energy at $K_i > X_i$ is:

$$\Delta E(\pi_{MAX,i}) = (Y_i - K_i/L)^* Q_{DC,i} - (Y_i - X_i/L)^* Q_{DC,i}$$

$$= -(K_i/L - X_i/L)^* Q_{DC,i}$$

This value represents the opportunity cost (via foregone profit) of discharging the cheapest available charging energy purchased at a realized value of X_i before reaching the next expected profit maximizing peak price hour.

Considering $\Delta E(\pi_{MAX,i})$ as the opportunity cost of generating positive output at this point on the expected price curve, and considering the negative of this value to properly consider as a cost, and noting that the cost of stored charging energy, or fuel cost for the ESR to generate quantity $Q_{DC,i}$, can be represented as $Q_{DC,i}$ /L*X_i, the general profit formation defined in earlier sections can be applied here for an interval of optimization sub-period i as:

$$\Pi(Q_{DC,i}) = P_i * Q_{DC,i} - [FC_i + Q_{DC,i}/L * X_i + (-\Delta E(\pi_{MAX,i}))]$$

$$= P_i * Q_{DC,i} - [FC_i + X_i/L * Q_{DC,i} + (K_i/L - X_i/L) * Q_{DC,i}]$$

$$= P_i * Q_{DC,i} - [FC_i + K_i/L * Q_{DC,i}]$$

Then the total cost of discharging on this portion of the expected price curve in an interval of optimization sub-period i is:

$$TC(Q_{DC,i}) = FC_i + K_i/L * Q_{DC,i}$$

Taking the first derivative of the total cost function TC with respect to $Q_{DC,i}$ yields MC, the marginal cost of discharging in an interval of optimization sub-period i at the specified point of the expected price curve:

$$\frac{\partial TC}{\partial Q} = MC = K_i/L$$

This amount is the cost of replacement charging energy at the next available opportunity, adjusted for losses, before reaching the next expected profit maximizing peak price hour[13].

[13] This will generally also be the next lowest cost charging energy available before the expected profit maximizing peak price. However, in cases where a non-profit maximizing peak occurs within the optimization sub-period, a less expensive charging opportunity may be available in the future. A bid that only considered this value for recharging opportunity may potentially miss out on additional profit before reaching that point. Thus the resource would optimally recharge at the next available opportunity.

CHARGING

Again consider the expected maximum profit for the optimization sub-period i as discussed above:

$$E(\pi_{MAX,\,i}) = E(REVENUE\pi_{MAX,\,i}) - E(COST\pi_{MAX,\,i})$$

$$= Q_{DC,i}{*}Y_i - Q_{CG,i}{*}X_i$$

If the resource is positioned to charge, the expected profit maximizing (least cost) charging opportunity in the optimization sub-period will be at expected price X_i . Similar to footnote 11 above regarding peak prices, in real-time expectation of X_i may be dynamic if prices are expected to be increasing.

If the ESR is positioned to charge because it has not yet realized the trough X_i associated with profit maximization (i.e., in the hour immediately preceding the expected profit maximizing trough), charging cost has not yet been realized and there is not sufficient time to again discharge before reaching the expected trough price for optimization sub-period i. Charging the ESR at price P_i occurring before reaching the expected trough hour of optimization sub-period i results in a fully foregone opportunity to purchase the lowest cost charging energy at X_i before discharge at the expected profit maximizing peak price Y_i . The resource would be indifferent to charging in the current hour versus the next hour of the expected trough for optimization sub-period i if the price in the current hour is at least as low as X_i expected the next hour. Possible occurrences of this scenario are summarized in Figures 6 and 7.

Figure 6

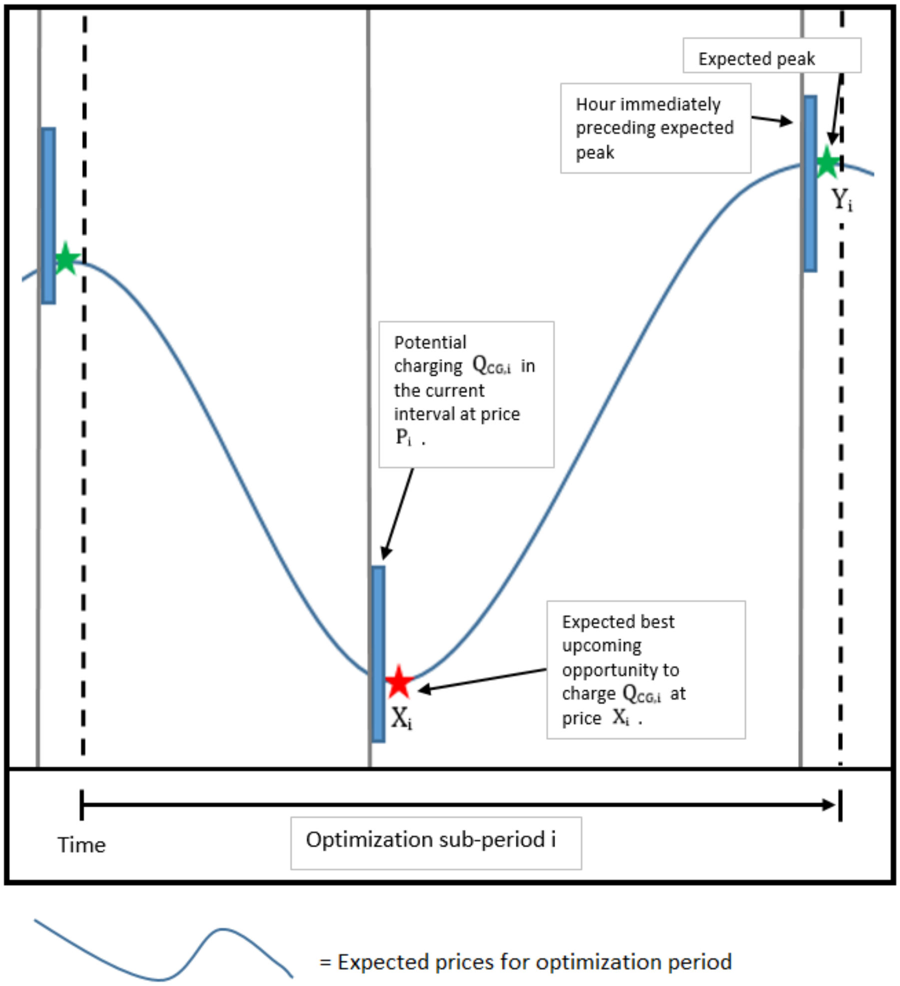
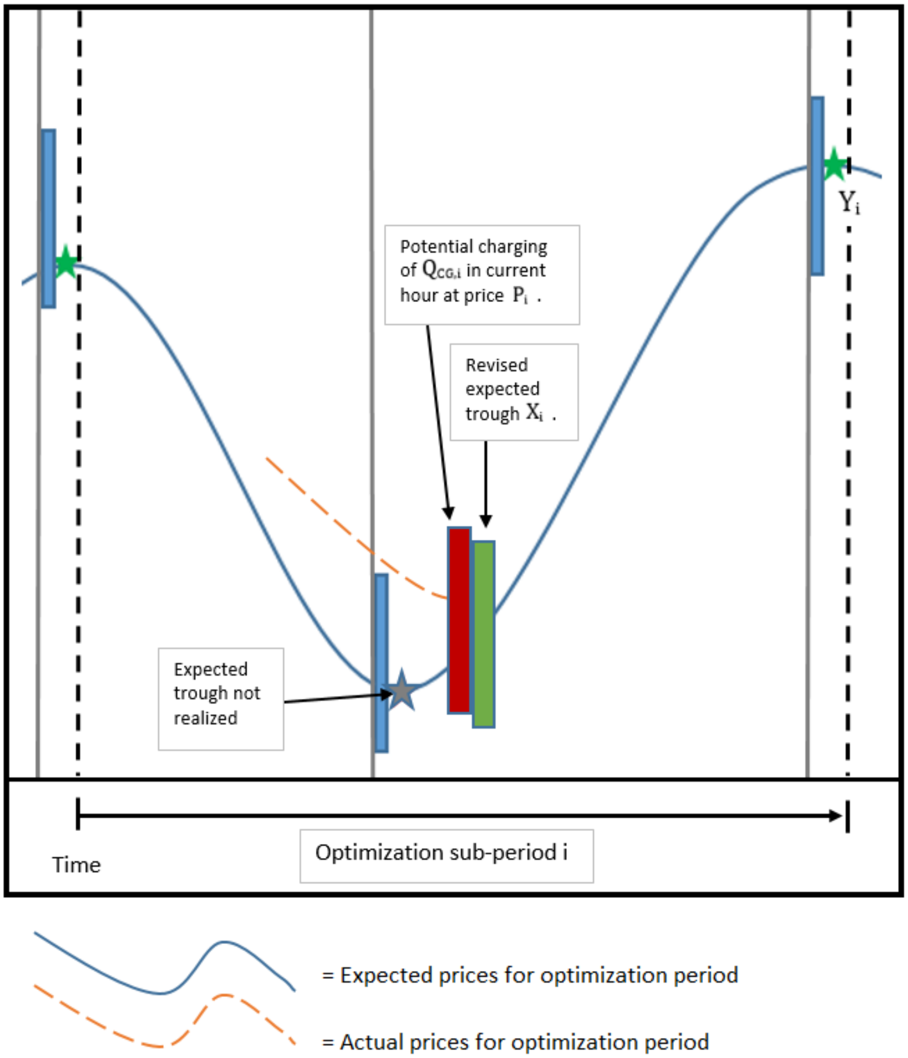

Expected peak
Hour immediately preceding expected peak
Yi
Potential charging QCG,i in the current interval at price Pi .
Expected best upcoming opportunity to charge QCG,i at price Xi .
Xi
Time
Optimization sub-period i
= Expected prices for optimization period

Figure 7



The value of the foregone charging opportunity for optimization sub-period i when the ESR instead charges at P_i can be represented as a change to the cost associated with producing the expected profit maximizing output:

$$\Delta E(COST\pi_{MAX,i}) = -X_i * Q_{CG,i}$$

This value is the amount by which cost associated with expected maximum profit would decrease for each unit that the ESR charges instead at the price occurring in the interval immediately before X_i.

Similar to the charging case presented above, the general profit form can be applied here. Again, because the production of charging energy reflects negative output of the resource, and to more easily contemplate decreases in charging as incremental generation, consider profit as a function of variable $D_{CG,i} = - Q_{CG,i}$. Also recall that a one unit increase in $D_{CG,i}$ is equivalent to a one unit reduction in charging or a one unit increase in generation. In the profit equation, revenue (negative or positive) is realized from producing charging energy, there is no variable cost from fuel, and the additional cost $\Delta E(COST\pi_{MAX,i})$ is realized if charging occurs at price P_i. Also as above, note that this value can be positive or negative.

Consider $\Delta E(COST\pi_{MAX,i})$ as OC_i in the production of charging energy. When defined as a function of $D_{CG,i}$ this value can be viewed as the opportunity cost of decreasing charging, equivalent to increasing generation. Then total profit realized from charging in an interval of sub-optimization period i is:

$$\Pi\,(D_{CG,i}) = P_i * D_{CG,i} - [FC_i + 0 + \Delta E(COST\pi_{MAX,\,i})\,]$$

$$= P_i * D_{CG,i} - [FC_i + X_i * D_{CG,i}\,]$$

The total cost of charging on the portion of the expected price curve in an interval of optimization sub-period i is:

$$TC\,(D_{CG,i}) = FC_i + X_i * D_{CG,i}$$

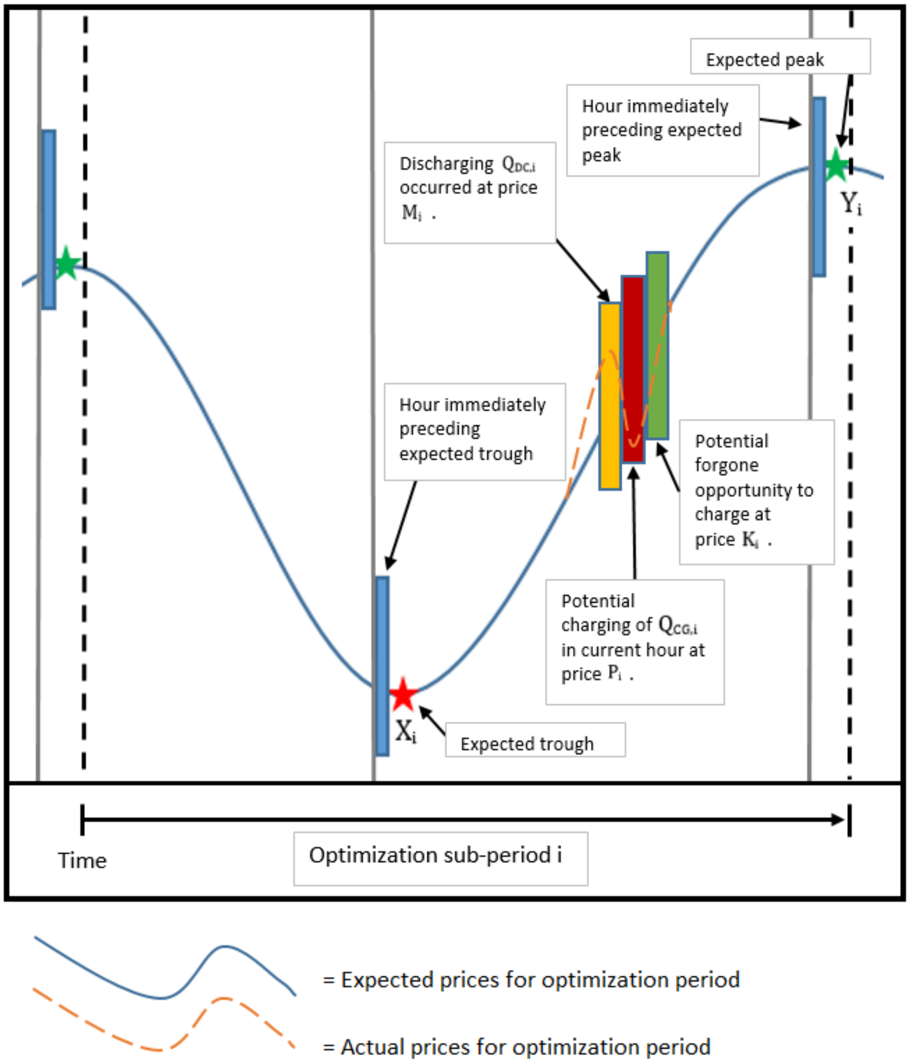
Taking the first derivative of the total cost function TC with respect to $D_{CG,i}$ yields MC , the marginal cost of a decrease in charging (or equivalently incremental generation) in an interval of optimization sub-period i at the specified point of the expected price curve:

$$\frac{\partial TC}{\partial D} = MC = X_i$$

Therefore the appropriate opportunity cost to include in the mitigated offer in the charging range in the hour immediately preceding the expected trough is X_i , the latest expectation of the trough in sub-period i. This value represents the cost that would be incurred to forgo charging (increment generation) in the current interval. In the case that the resource has not yet charged for optimization sub-period i, the result is the latest expectation of the profit maximizing trough, X_i. This is the next expected price K_i, so this scenario is the case of $K_i = X_i$. However, the result more generally is the expected price in the next hour, K_i.

Consider now the case where the ESR is positioned to charge on this portion of the expected price curve because of an economic discharge in an earlier interval of optimization sub-period i that occurred at price M_i occurring before the current interval with price P_i. A charge is required to position the ESR to realize the profit maximizing discharge price Y_i. If the ESR charges in the current interval at price P_i, the next best expected opportunity to charge in the next interval at K_i is foregone. This situation is summarized in Figure 8.

Figure 8



Then the cost associated with the expected profit maximizing charging changes as follows:

$$\Delta E(COST\pi_{MAX,i}) = X_i{*}Q_{CG,i} - M_i{*}Q_{DC,i} - (X_i{*}Q_{CG,i} - M_i{*}Q_{DC,i} + K_i{*}Q_{CG,i})$$

$$= - K_i{*}Q_{CG,i}$$

This value is the amount by which cost associated with expected maximum profit would decrease for each unit that the ESR charges in the current interval instead of the expected price in the next interval K_i.

Similar to the charging cases presented above, the general profit form can be applied here. Again, because the production of charging energy reflects negative output of the resource, and to more easily contemplate decreases in production as incremental generation, consider profit as a function of variable $D_{CG,i} = - Q_{CG,i}$. Also recall that a one unit increase in $D_{CG,i}$ is equivalent to a one unit reduction in charging or a one unit increase in generation. In the profit equation, revenue (negative or positive) is realized from producing charging energy, there is no variable cost from fuel, and the change in cost $\Delta E(COST\pi_{MAX,i})$ is realized if charging occurs at price P_i. Also as above, note that this value can be positive or negative.

Consider $\Delta E(COST\pi_{MAX,i})$ as OC_i in the production of charging energy. When defined as a function of $D_{CG,i}$ this value can be viewed as the opportunity cost of decreasing charging, equivalent to increasing generation.

Then total profit realized from charging in an interval of sub-optimization period i is:

$$\Pi\ (D_{CG,i}) = P_i * D_{CG,i} - [FC_i + 0 + \Delta E(COST\pi_{MAX,\,i})\ ]$$

$$= P_i * D_{CG,i} - [FC_i + K_i * D_{CG,i}\ ]$$

The total cost of charging on the portion of the expected price curve in an interval of optimization sub-period i is:

$$TC\ (D_{CG,i}) = FC_i + K_i * D_{CG,i}$$

Taking the first derivative of the total cost function TC with respect to $D_{CG,i}$ yields MC, the marginal cost of a decrease in charging (or equivalently incremental generation) in an interval of sub-optimization period i at the specified point of the expected price curve:

$$\frac{\partial TC}{\partial D} = MC = K_i$$

As in the other charging case on this portion of the expected price curve, the marginal opportunity cost associated with a decrease in charging energy to be included in the mitigated energy offer is the expected price in the next hour, K_i.

SECTION 4: LAST HOUR OF OPTIMIZATION PERIOD

During the last hour of the optimization period, there is no opportunity cost to discharge and the basis of the mitigated discharge offer becomes the charging energy cost to produce 1 MW, X_i/L. This result is consistent with the above results in the absence of opportunity costs. The result is also is consistent with a special case of a more general marginal cost equation for compressed air energy storage presented in [2] that does not consider opportunity cost. If the resource is positioned to charge in the last hour, it expects profit of $0 and therefore has no opportunity cost, and there are no fuel costs associated with charging. Therefore the appropriate basis for a mitigated charging offer for this hour would be $0 to reflect the lack of fuel or opportunity cost associated with charging in this interval. Note that this case for a mitigated offer may apply more generally to instances where there is little to no opportunity cost. For example, depending on state-of-charge at the beginning of the optimization period, this case could apply to some resources that have long charging or discharging times that approach or exceed the length of the optimization period.

SECTION 5: SUMMARY TABLE

Point Within Period to Optimize[14,15]	Potential Basis for Mitigated Energy Offer – Discharge	Potential Basis for Mitigated Energy Offer - Charge
Hour immediately preceding expected profit maximizing peak or expected prices moving toward trough associated with profit maximization	K_i , expected price in the next hour	K_i*L, expected price in the next hour times roundtrip efficiency loss factor
Hour immediately preceding expected trough associated with profit maximization or expected prices moving toward expected profit maximizing peak	K_i/L, expected price in the next hour divided by roundtrip efficiency loss factor	K_i, expected price in the next hour
Last hour of optimization period, or more generally in cases of no opportunity cost	X_i/L, marginal cost of charging energy	$0

[14] The summary table provides an implementable approach to calculate the basis of an ESR mitigated energy offer that includes opportunity cost. However, when the expected price in the next hour is negative, the mitigated offer for the discharging range will be less than the mitigated offer for the charging range. The outcome is expected because losses create additional revenue when prices are negative. However, when the ESR is modeled with one continuous offer curve, the resulting mitigated offer curve is not monotonically non-decreasing. Adjustments such as multi-configuration modeling, or simplifications of the approach when negative prices are expected, must be made to ensure monotonic non-decreasing offer curves.

[15] For cases where the expected profit maximizing peaks and troughs occur in immediate succession, with no other intervals to charge or discharge in between, the mitigated offer basis becomes K_i, the expected price in the next hour, for both charging and discharging. This reflects the fact that there are no additional opportunities to charge or discharge before reaching the expected profit maximizing operation points.

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